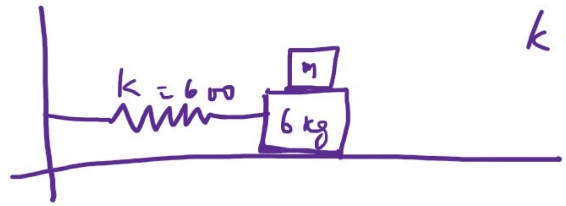



$k =$

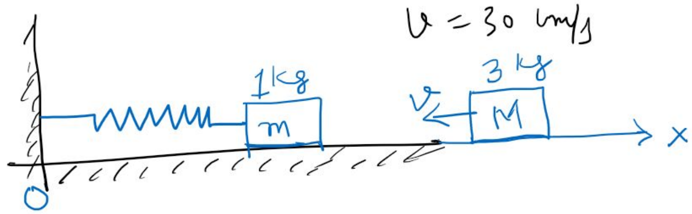
$T = 0.75$ second

Block 'm' will not slip relative to cart, if the cart is displaced 50 mm

$$T = 2\pi\sqrt{\frac{m+6}{k}} \Rightarrow \frac{T^2 k}{4\pi^2} - 6 = m$$

$$\therefore\ f \geq m\omega^2 A \Rightarrow \mu_s mg \geq m\omega^2 A \Rightarrow \boxed{\mu \geq \frac{\omega^2 A}{g}}$$

$$\therefore\ \mu \geq \left(\frac{2\pi}{T}\right)^2 \cdot \left(\frac{A}{g}\right)$$




$x = 10 + 3\sin(10t)$ cm. $\Rightarrow$ at $t=0$ $\quad x = 10$ cm.

$\omega = 10$

$v = \frac{dx}{dt} = 30\cos(10t)$

$\boxed{M}$ collides with $\boxed{m}$, performing SHM, at $t=0$ and get stuck to it.

By Conservation of Momentum :- $(\overset{3}{M}+\overset{1}{m})v' = \overset{3}{M}(30) - \overset{1}{m}(30) \Rightarrow (4)v' = 60$

$v' = 15$ cm/s $(-\hat{i})$

$K = m\omega^2 = 1\times 10^2 = 100\ \frac{N}{m}$

$A' = \sqrt{\frac{M+m}{K}} \cdot v' = 0.03$

$\omega' = \sqrt{\frac{K}{M+m}} = 5\ \frac{rad}{s}$

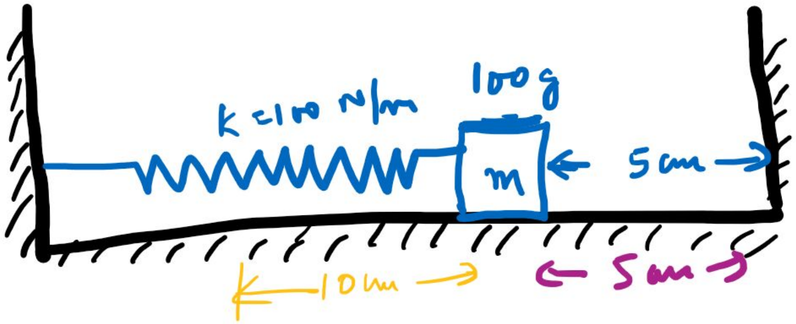
Initially

at $t=0$ 'm' is at mean position ($x = 10$ cm)

$\Rightarrow v_{max} = \omega A = 30\,(\hat{i})$ cm/s $\Rightarrow A = 3$

$\therefore \Delta KE = \frac{1}{2}(1)(0.3)^2 + \frac{1}{2}(3)(0.3)^2 - \frac{1}{2}(4)(0.15)^2 = \underline{0.135\ J}$

$\therefore x' = 10 - 3\sin(5t)$ cm ✓




Block is moved to compress the spring by 10 cm and released

(#) Consider the collisions with the wall are elastic

$$\frac{A}{2} = A\sin\left(\frac{2\pi}{T}\cdot t\right)$$
$$\frac{\pi}{6} = \frac{2\pi}{T}\cdot t$$
$$\Rightarrow t = \frac{T}{12}$$

Time period of oscillation : $T = t_{RHS} + t_{LHS}$

$$= \left(\frac{T}{2}\right) + 2\left(\frac{T}{12}\right)$$

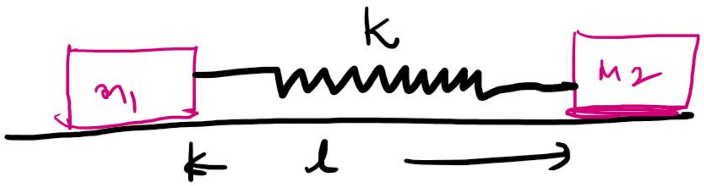
$$= \frac{T}{2} + \frac{T}{6}$$

$$= \frac{4T}{6} = \frac{2}{3}T = \frac{2}{3}\times 2\pi\sqrt{\frac{m}{k}}$$

$$= \frac{4\pi}{3}\sqrt{\frac{m}{k}}$$

$$\therefore T = \frac{4\pi}{3}\sqrt{\frac{0.1}{100}}$$
$$= \frac{4\pi}{3}\times\frac{1}{10\sqrt{10}} = 0.133$$

Compound oscillator [ two body oscillator system ]




$\mu$ = reduced mass

$$T = 2\pi\sqrt{\frac{\mu}{k}}, \quad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

If the spring is elongated by '$x_0$' and released

Blocks oscillate with same time period but different amplitudes

As $A \propto \frac{1}{m}$

$$\frac{A_1}{A_1} = \frac{m_2}{m_1}$$

(8): $A_1 + A_2 = x_0$ and $m_1 A_1 = m_2 A_2$

$$\Rightarrow \quad A_1 = \left(\frac{m_2}{m_1 + m_2}\right) x_0$$

$$A_2 = \left(\frac{m_1}{m_1 + m_2}\right) x_0$$

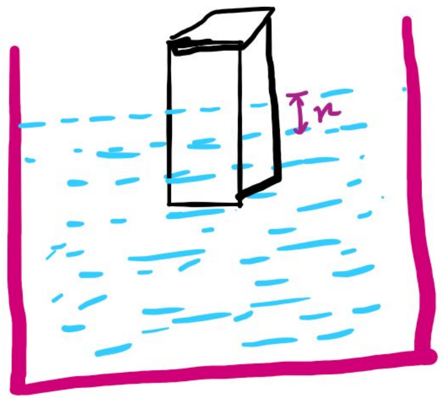
To Calculate 'T' of a oscillations of a floating body

$m$ = mass of the body

$A$ = Area of C.S

$\rho$ = density of liquid

$\sigma$ = density of the body




When the body is displaced by small displacement '$x$'

Net Restoring force $= -$ [change in variable force (upthrust)]

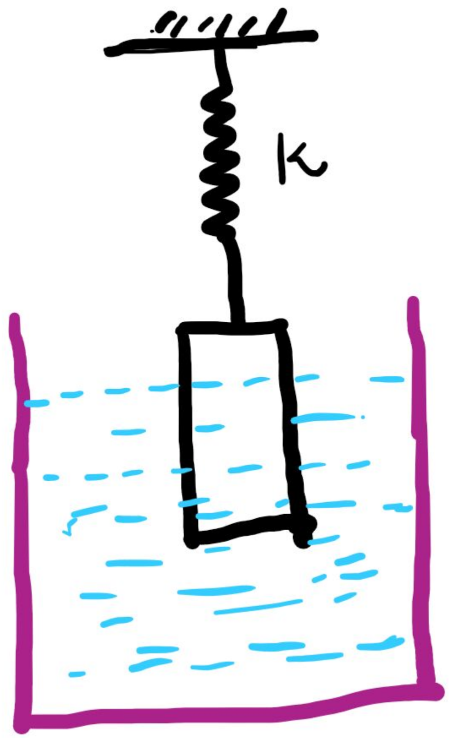
$F_R = -$ [extra immersed volume $\times$ density of liquid $\times g$]

$$-kx = -\left[(Ax)\rho g\right] \Rightarrow k = A\rho g$$

$$\therefore \; T = 2\pi\sqrt{\frac{m}{A\rho g}} \quad (\text{or}) \quad f = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$$

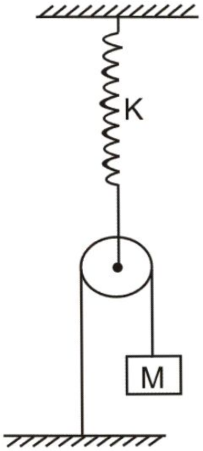
Find Time period of the body [for small displacement]

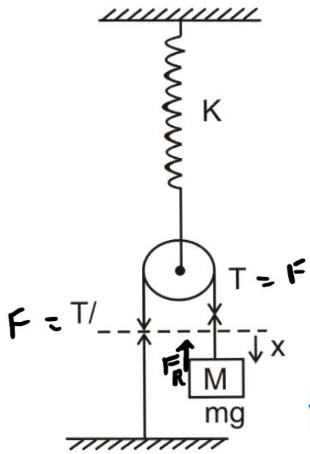
in the following situation




here $K_{effective} = K + A\rho g$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{K + A\rho g}}$$




for small displacement of Block 'x'

Spring extends by $x_0 = \frac{x}{2}$

$2F = kx_0$

$x_0 = \frac{2F}{k} = \frac{x}{2}$

$\therefore \; F = \frac{k}{4} \cdot x$

Effective $k = \frac{k}{4}$

$\therefore$ Restoring force $= F_R = -\frac{k}{4}x = ma$

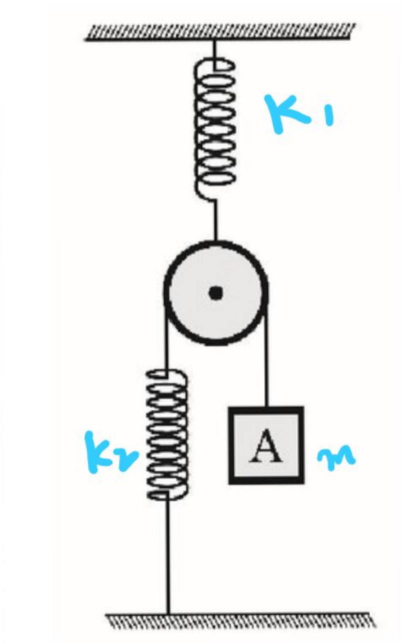
$\therefore \; a = -\frac{k}{4m} \cdot x$

Comparing with $a = -\omega^2 x$

$\left.\begin{array}{l} a = -\frac{k}{4m}\cdot x \\ a = -\omega^2 x \end{array}\right\} \; \omega = \sqrt{\frac{k}{4m}}$

$\therefore \; T = 2\pi\sqrt{\frac{m}{(k/4)}} = 2\pi\sqrt{\frac{4m}{k}}$

Find 'T' for small displacement of the **Block**:

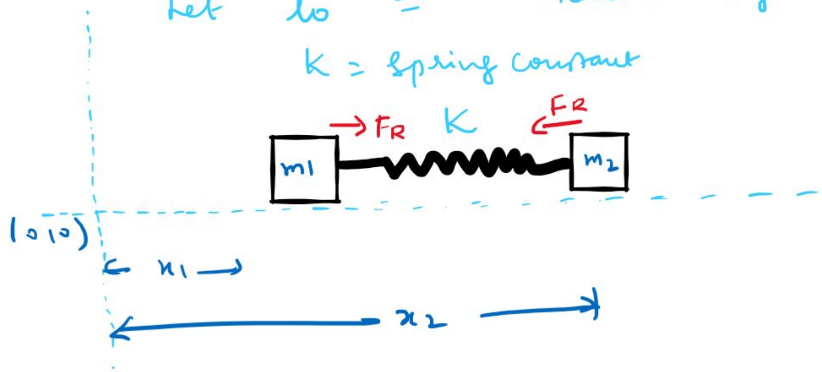



$$K_{eff} = \frac{\left(\frac{k_1}{4}\right)(k_2)}{\left(\frac{k_1}{4}\right)(k_2)}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eff}}}$$

Let $l_0$ = natural length of spring

$K$ = spring constant




Let $x_1$ and $x_2$ be the co-ordinates of $m_1$ and $m_2$ at any instant of time.

$\therefore$ extension of spring $= x = [x_2 - x_1] - l_0$

$$\frac{d^2x}{dt^2} = \frac{d^2}{dt^2}[x_2 - x_1]$$

acceleration

$\therefore$ For $x > 0$, $\quad m_1 \times \left(m_2 \dfrac{d^2x_2}{dt^2}\right) = (-kx) \times m_1$

$$m_2 \times \left(m_1 \frac{d^2x_1}{dt^2}\right) = +kx \times m_2$$

We get

$$m_1 m_2 \frac{d^2x_2}{dt^2} - m_1 m_2 \frac{d^2x_1}{dt^2} = -[m_1 + m_2]kx$$

$$\therefore \left(\frac{m_1 m_2}{m_1 + m_2}\right) \frac{d^2}{dt^2}[x_2 - x_1] = -kx$$

$$\Rightarrow a = -\frac{K}{\mu} \cdot x$$

where $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$

Hence

$$T = 2\pi \sqrt{\frac{\mu}{K}}$$